Perturbation de période due au développement excentrique du spiral

Spiral plat sans courbes terminales

Balancier annulaire monométallique d'une montre bracelet

Référence :D:\Résonateur (TE)/Data\Montre HES.mcd(R)

$r_1 := \frac{d1_{sp}}{2}$ $\quad r_1 = 0.55\,mm$ $\quad r_2 := \frac{d2_{sp}}{2}$ $\quad r_2 = 2.26\,mm$ $\quad n_{sp} = 12.667$ $\quad p_{sp} = 0.135\,mm$

$a_{sp} := \frac{p_{sp}}{2\cdot\pi}$ $\quad spire := 2\cdot\pi$ $\quad \alpha_1 := \frac{r_1}{a_{sp}}$ $\quad \alpha_2 := \frac{r_2}{a_{sp}}$ $\quad s(\alpha) := \int_{\alpha_1}^{\alpha} a_{sp}\cdot\sqrt{1+\alpha^2}\,d\alpha$ $\quad r(\alpha) := a_{sp}\cdot\alpha$

$\alpha_1 = 4.07\,spire$ $\quad \alpha_2 = 16.74\,spire$ $\quad \psi := \alpha_2 - \alpha_1$ $\quad \psi = 12.67\,spire$ $\quad L := s(\alpha_2)$ $\quad L = 11.183\,cm$

$T_0 = 0.25\,s$ $\quad f = 4\,s^{-1}$ $\quad \omega_0 := 2\cdot\pi\cdot f$ $\quad \theta_0 = 270\,deg$ $\quad h_{déc} := 0.2\cdot mm$ $\quad \beta_{déc} := 20\cdot deg$

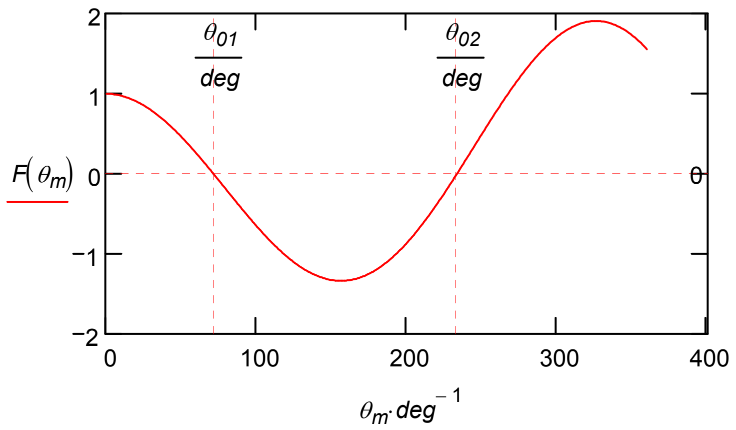
Spiral non déformé en position de repos

$F(\theta_0) := J0(\theta_0) - \theta_0\cdot J1(\theta_0)$ $\quad F(\theta_0) = 1.061$

$Delta(\theta_0) := \frac{2}{L^2}\cdot\frac{1}{r_2^2 + r_1^2}\cdot\left[-\left(r_2^4 + r_1^4\right) + 2\cdot r_2^2\cdot r_1^2\cdot F(\theta_0)\cdot cos\right.$ $\quad Delta(\theta_0) = -8.223\times 10^{-}$ $\quad A := \frac{2}{L^2}\cdot\frac{r_2^4 + r_1^4}{r_2^2 + r_1^2}$

$\mu(\theta_0) := -86400\cdot Delta(\theta_0)$ $\quad \boxed{\mu(\theta_0) = 71.044}$ $\quad \boxed{\mu(220\cdot deg) = 65.315}$

$x := 100\cdot deg$ $\quad \theta_{01} := racine(F(x), x)$ $\quad \xi := 300\cdot deg$ $\quad \theta_{02} := racine(F(\xi), \xi)$ $\quad \theta_m := 1\cdot deg, 2\cdot deg\,..\,360\cdot deg$



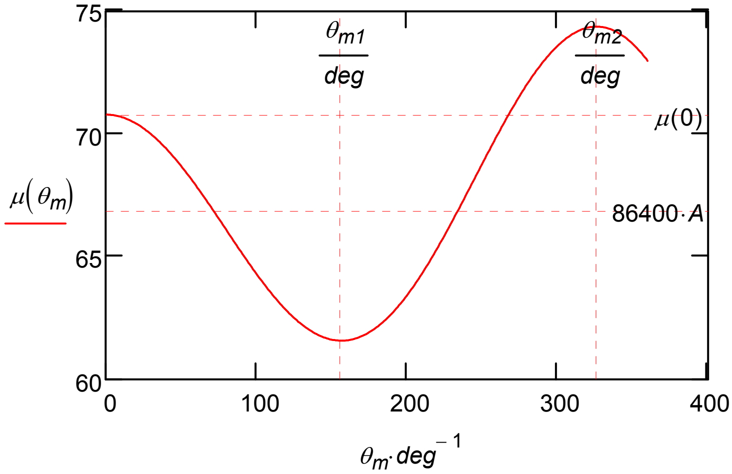
$\theta_{01} = 72\,deg$

$\theta_{02} = 233.7\,deg$

$\theta_{m1} := racine\left(\frac{d}{dx}\mu(x), x\right)$

$\theta_{m1} = 156.7\,deg$

$\theta_{m2} := racine\left(\frac{d}{d\xi}\mu(\xi), \xi\right)$



$\theta_{m2} = 326.1\,deg$

$\mu_{m1} := -86400 \cdot Delta(\theta_{m1})$

$\mu_{m1} = 61.585$

$\mu_{m2} := -86400 \cdot Delta(\theta_{m2})$

$\mu_{m2} = 74.383$

Spiral déformé en position de repos

$\delta_1(\theta_0) := Delta(\theta_0)$ $\qquad$ $\delta_1(\theta_0) = -8.223 \times 10^{-4}$

$$\delta_2(\theta_0, h) := \frac{-4 \cdot h^2}{r_2{}^2 + r_1{}^2} \cdot \frac{J1(\theta_0)}{\theta_0}$$

$\delta_2(\theta_0, h_{déc}) = 1.768 \times 10^{-3}$

$$\delta_3(\theta_0, h, \beta) := -\frac{4 \cdot h}{\left(r_2{}^2 + r_1{}^2\right) \cdot L} \cdot \left(r_2{}^2 \cdot \sin(\alpha_2 - \beta) - r_1{}^2 \cdot \sin(\alpha_1 - \beta)\right) \cdot J0(\theta_0)$$

$\delta_3(\theta_0, h_{déc}, \beta_{déc}) = -1.661 \times 10^{-3}$

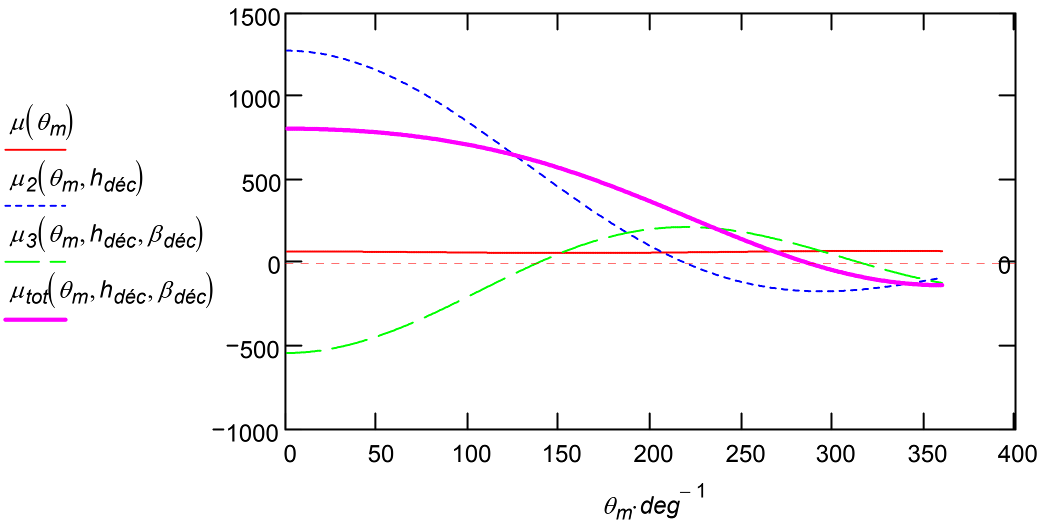
$\delta_{tot}(\theta_0, h, \beta) := \delta_1(\theta_0) + \delta_2(\theta_0, h) + \delta_3(\theta_0, h, \beta)$ $\qquad$ $\delta_{tot}(\theta_0, h_{déc}, \beta_{déc}) = -7.156 \times 10^{-4}$

$\mu_2(\theta_0, h) := -86400 \cdot (\delta_2(\theta_0, h))$ $\qquad$ $\boxed{\mu_2(\theta_0, h_{déc}) = -152.725}$

$\mu_3(\theta_0, h, \beta) := -86400 \cdot (\delta_3(\theta_0, h, \beta))$ $\qquad$ $\boxed{\mu_3(\theta_0, h_{déc}, \beta_{déc}) = 143.506}$

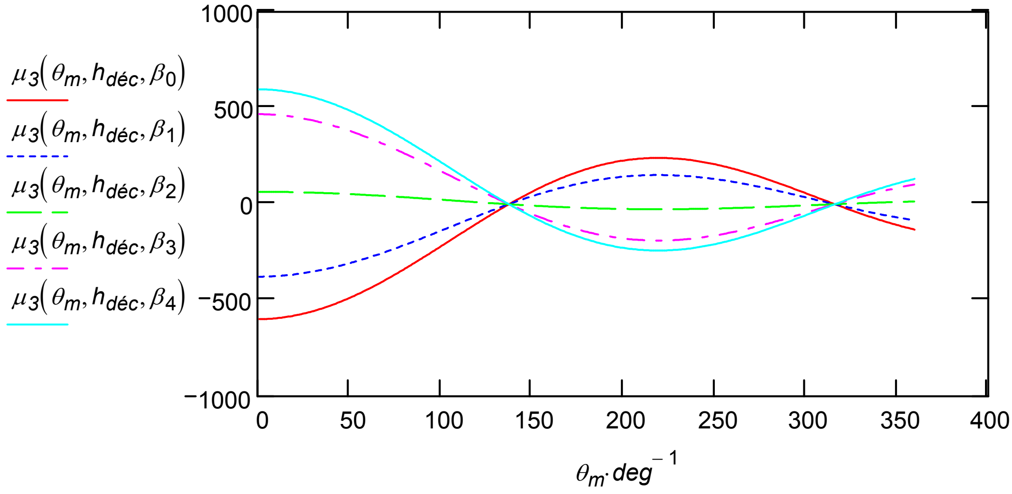
$\mu_{déc}(\theta_0, h, \beta) := -86400 \cdot (\delta_2(\theta_0, h) + \delta_3(\theta_0, h, \beta))$ $\qquad$ $\boxed{\mu_{déc}(\theta_0, h_{déc}, \beta_{déc}) = -9.218}$

$\mu_{tot}(\theta_0, h, \beta) := -86400 \cdot (\delta_1(\theta_0) + \delta_2(\theta_0, h) + \delta_3(\theta_0, h, \beta))$ $\qquad$ $\boxed{\mu_{tot}(\theta_0, h_{déc}, \beta_{déc}) = 61.825}$



Influence de la position angulaire du décentrage initial

$i := 0, 1 .. 4$ $\qquad$ $\beta_i := 45 \cdot deg \cdot i$

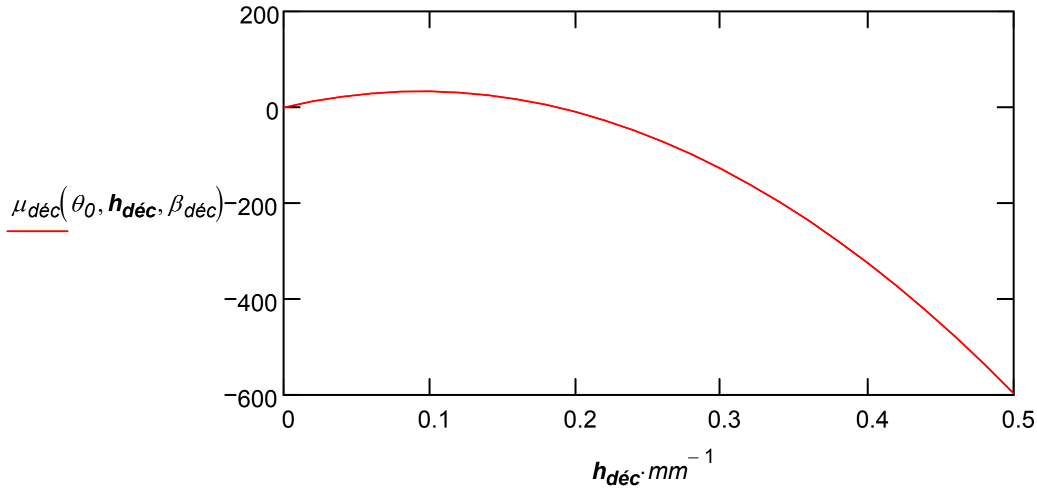


$h_{déc} = 0.2\,mm$

$\theta_0 = 270\,deg$

Influence de la position radiale du décentrage initial

$\mathbf{h_{déc}} := 0 \cdot mm, .02 \cdot mm .. 0.5mm$ $\qquad \beta_{déc} = 20\,deg$ $\qquad \theta_0 = 270\,deg$



Vérification par calcul numérique

$$\theta(\varphi) := \theta_0 \cdot \cos(\varphi)$$

$$z(\alpha) := a_{sp} \cdot \alpha \cdot \exp(i \cdot \alpha) \qquad \sigma^2 = \sqrt{\frac{1}{L} \cdot \int_{\alpha_1}^{\alpha_2} (|z(\alpha)|)^2 \cdot a_{sp} \cdot \alpha \, d\alpha} \qquad \sigma := \sqrt{\frac{1}{2} \cdot \left(r_2^{\,2} + r_1^{\,2}\right)} \qquad \sigma = 1.645 \times 10^{-3}\,m$$

$$\Delta_1(\theta) := i \cdot \frac{a_{sp}^{\,2} \cdot \theta}{L} \cdot \exp(i \cdot \theta) \cdot \int_{\alpha_1}^{\alpha_2} \alpha^2 \cdot \exp(i \cdot \alpha) \cdot \exp\left[-i \cdot \frac{\theta \cdot a_{sp} \cdot \left(\alpha^2 - \alpha_1^{\,2}\right)}{2 \cdot L}\right] d\alpha$$

$$\Delta_1(\theta) := \theta \cdot \frac{a_{sp}^{\,2}}{L} \cdot \exp(i \cdot \alpha_1) \cdot \left(\alpha_2^{\,2} \cdot \exp(i \cdot \psi) - \alpha_1^{\,2} \cdot \exp(i \cdot \theta)\right)$$

$$\Delta_2(\theta, h, \beta) := i \cdot \frac{\theta}{L} \cdot \exp(i \cdot \theta) \cdot \int_0^L h \cdot \exp(i \cdot \beta) \cdot \exp\left(-i \cdot \theta \cdot \frac{s}{L}\right) ds \qquad \Delta_2(\theta, h, \beta) := -h \cdot \exp(i \cdot \beta) \cdot (1 - \exp(i \cdot \theta))$$

$$\Delta(\theta) := \Delta_1(\theta) + \Delta_2(\theta, h_{déc}, \beta_{déc})$$

$$\chi(\theta) := \frac{\Delta(\theta)\cdot\overline{\Delta(\theta)}}{\sigma^2} \qquad \chi(\theta_0) = 0.085 \qquad \gamma(\theta) := \frac{d}{d\theta}\chi(\theta) \qquad Gamma(\varphi) := \gamma(\theta(\varphi))$$

$$\delta_{num} := \frac{-1}{2\cdot\pi\cdot\theta_0^{\,2}}\cdot\int_0^{2\cdot\pi}\theta(\varphi)\cdot Gamma(\varphi)\,d\varphi \qquad \boxed{\delta_{num} = -7.156\times10^{-4}} \qquad \delta_{tot}(\theta_0, h_{déc}, \beta_{déc}) = -7.156\times10^{-4}$$

$$\mu_{num} := -86400\cdot\delta_{num} \qquad \boxed{\mu_{num} = 61.825} \qquad \mu_{tot}(\theta_0, h_{déc}, \beta_{déc}) = 61.825$$